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Prediction of Turbulent Jet Mixing Noise Reduction by Water Injection

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A one-dimensional control volume formulation is developed for the determination of jet mixing noise reduction due to water injection. The analysis starts from the conservation of mass, momentum, and energy for the control volume and introduces the concept of effective jet parameters (jet temperature, jet velocity, and jet Mach number). It is shown that the water-to-jet mass flow rate ratio is an important parameter characterizing the jet noise reduction on account of gas-to-droplet momentum and heat transfer. Two independent dimensionless invariant groups are postulated and provide the necessary relations for the droplet size and droplet Reynolds number. Results are presented illustrating the effect of mass flow rate ratio on the jet mixing noise reduction for a range of jet Mach numbers and jet Reynolds numbers. Predictions from the model show satisfactory comparison with available test data on perfectly expanded hot supersonic jets. The results suggest that significant noise reductions can be achieved at increased flow rate ratios.

Nomenclature

A	=	cross-sectional area of the jet
B	=	Spalding transport number, $h_{fg}/\lfloor c_{pj}(T_{j1} - T_{\text{sat}})\rfloor$
C_D	=	drag coefficient, $F_D/[\frac{1}{2}(\rho_j u_j^2)(\pi d_p^2/4)]$
c	=	sound velocity
c_p	=	specific heat at constant pressure
c_1, c_2, c_3	=	invariants (dimensionless)
d	=	diameter
F	=	force
f	=	frequency
h_{fg}	=	latent heat of evaporation
k	=	thermal conductivity of gas
M_j	=	jet Mach number
M_{je}	=	fully expanded jet Mach number
$\dot{m}_j$	=	jet mass flow rate
$\dot{m}_w$	=	component of mass flow rate of water in the jet axial direction
$\dot{m}_{wt}$	=	total injected mass flow rate of water
N_p	=	droplet number per unit volume of the mixture
Nu	=	droplet Nusselt number
Pr	=	Prandtl number of gas, $c_{pg}\mu_g/k_g$
p	=	pressure
q_p	=	heat of evaporation of the droplet
R	=	gas constant
Re_{j1}	=	jet exit Reynolds number, $\rho_{j1}u_{j1}d_{j1}/\mu_{j1}$
Re_p	=	droplet Reynolds number, $\rho_{j1}\lvert u_{j1} - u_p\rvert d_p/\mu_{j1}$
T	=	temperature
T_{je}	=	equivalent gas temperature for heat transfer
u	=	velocity in the streamwise direction
x	=	axial distance from the nozzle exit plane
x_t	=	length of potential core in the jet
α	=	angle of water injection (measured from the downstream jet axis)
γ	=	isentropic exponent
η	=	fraction of injected water flow rate that is evaporated
μ	=	dynamic viscosity
ρ	=	density

Subscripts

a	=	ambient fluid
cr	=	critical
D	=	drag
g	=	gas
j	=	jet
l	=	liquid
p	=	droplet (or particle)
sat	=	saturation
t	=	stagnation
w	=	injected water
1	=	jet exit
2	=	effective jet property

I. Introduction

THREE distinct components of noise are present in supersonic jets: turbulent mixing noise, Mach wave radiation, and broadband shock noise (see Crighton [1]). Whereas the broadband shock noise (emanating from shock–turbulence interaction) is associated with imperfectly expanded jets, the turbulent mixing noise generally represents the dominant component of supersonic jet noise to the overall sound pressure level (OASPL), with the Mach wave radiation (caused by the supersonic convection of turbulent eddies with respect to the ambient fluid) contributing significant noise in the peak-radiation direction. The high noise levels (160 to 170 dB) radiated by launch vehicles at liftoff induce severe vibration on the launch vehicle structure, payload, and ground support equipment.

Water injection has been traditionally considered for the suppression of high noise levels from rocket exhausts in launch vehicle environments. For example, large amounts of water (of the order of 300,000 gpm) are used for the suppression of ignition overpressure and liftoff noise during space shuttle launches. The ratio of the water mass flow rate to the solid rocket booster exhaust mass flow rate is maintained near one or two to meet payload design requirements of 145 dB (Dougherty and Guest [2] and Jones [3]). Water injection could reduce noise by as much as 8–12 dB [2,3]. Such a high level of reduction includes reductions in the turbulent

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mixing noise and the broadband shock noise, the latter representing the predominant component of noise reduction.

Water injection mitigates all three components of jet noise: the turbulent mixing noise, the Mach wave radiation, and the broadband shock noise. Two principal mechanisms leading to the diminution of jet noise by water injection are the reduction of jet velocity and jet temperature (Moriniere et al. [4]). The decrease of jet velocity is occasioned through momentum transfer between the liquid and the gaseous phases, and the reduction of the jet temperature is achieved due to partial vaporization of the injected water (Zoppellari and Juve [5,6]). The effect of water may also be regarded as effectively increasing the effective jet density (Jones [3]). Important velocity reductions are achieved within a few diameters of the nozzle exit. Noise reductions of the order of 10 dB are realized for both cold and hot jets [5,6].

Several design parameters influence the effectiveness of noise reduction by water injection. These include water-to-jet mass flow rate ratio, axial injection location, water-injection angle, number of injectors, method of injection (jet type or spray type), droplet size, water pressure, and water temperature. Optimal injection parameters need to be determined for the design of an efficient water-injection system. Data of Zoppellari and Juve [5] and of Norum [7] suggest that the best noise reductions of the order of 10 to 12 dB are obtained at injection angles of 45 to 60 deg, injection near the nozzle exit (especially for shock-containing jets), and high mass flow rates. The optimum number of injectors appears to be around eight. Experiments by Krothapalli et al. [8], Greska and Krothapalli [9], and Washington and Krothapalli [10] at reduced water mass flow rate ratios (about 0.1) through the use of microjets show sizable noise reduction for application to aircraft jet engines. The noise reduction with microjet injection is shown to be achieved primarily through reduced turbulence in the carrier gas, whereas the mean velocity field remains essentially unaltered.

Experiments with water injection suggest that the mass flow rate ratio appears to be an important parameter. Tests conducted with water-to-jet mass flow rate ratios up to four (Zoppellari and Juve [5]) reveal that significant noise reductions can be achieved at high water flow rate ratios. In the case of cold jets, beyond a critical mass flow rate ratio, the velocity reduction (and thus the noise reduction) is small. For hot jets, only a fraction of the liquid is effective at reducing the air jet velocity due to drop evaporation. At low water flow rates, it is possible to reduce the shock-associated noise significantly.

At considerably high mass flow rates, the benefit of velocity reduction of the air jet by momentum transfer between the two phases is partly opposed by the emergence of new parasitic sources linked to water injection, which include the impact noise of air on the water jets, fragmentation of these water jets, and unsteady movement of the droplets. A compromise can be found between significant penetration of the water jet into the air jet and low impact noise. A significant parameter is the velocity component of water jets that is perpendicular to the air jet. If this component is high, water penetrates deeply into the air jet and mixing takes place rapidly. If this component is small, water does not produce significant drag and impact noise.

On the theoretical front, studies on the effect of water droplets on noise reduction are very few, and these are concerned primarily with plane waves. Sound-attenuation studies in a two-phase medium originated from the work of Sewell [11] on the assumption of immovable particles. Epstein and Carhart [12] considered sound attenuation by particles, allowing the particles to oscillate and considering both momentum and heat exchange. They compared the theory with the measurements with water droplets by Knudsen et al. [13]. Marble and Wooten [14] and Marble and Candel [15] conducted theoretical investigations to determine the sound attenuation in fans and ducts by vaporization of liquid droplets. For plane waves (Marble and Candel [15]), the attenuation magnitude is shown to exceed 5 dB/m at 25 C with a cloud of 0.7-μm-radius droplets constituting 1% of the gas mixture (see Krothapalli et al. [16]). Temkin and Dobbins [17] presented a classical study of both attenuation and dispersion of sound by spherical droplets, considering the particulate relaxation processes of momentum and heat transport. Dupays and Vuillot [18] extended the analysis of Temkin and Dobbins [17] to accommodate the effects of droplet evaporation and combustion. Unfortunately, these results are not directly applicable to the study of turbulent mixing noise attenuation in jets.

In view of the importance of water injection in jet noise suppression, a theoretical understanding of the mechanism of noise reduction is useful in the design and optimization of water-injection systems for launch acoustics application. To the author's knowledge, theoretical studies addressing the effect of water injection in mitigating jet mixing noise have not been previously reported. It is the purpose of this paper to develop a simple one-dimensional analytical model for estimating the jet mixing noise suppression due to water injection. The proposed model will be compared with existing data on jet noise. This paper is based primarily on [19].

II. Analysis

A. Control Volume Formulation and Conservation Equations

The governing equations of continuity, momentum, and energy for the control volume (Fig. 1) are expressed as

$$\rho_{j1} u_{j1} A_{j1} = \rho_{j2} u_{j2} A_{j2} - \eta \dot{m}_w \tag{1}$$

$$\rho_{j2} u_{j2}^2 A_{j2} - \rho_{j1} u_{j1}^2 A_{j1} = -F_d + \eta \dot{m}_w u_p \tag{2}$$

$$\dot{m}_{j2}\left(c_{pj} T_{j2} + \frac{u_{j2}^2}{2}\right) = \dot{m}_{j1}\left(c_{pj} T_{j1} + \frac{u_{j1}^2}{2}\right) - F_d u_p + \eta \dot{m}_w T_p c_{pl} \tag{3}$$

In the preceding equations, $\dot{m}_w$ is the mass flow rate of water in the jet axial direction, η is the fraction of the injected water flow rate that is evaporated, F_d is the drag force of droplets, u_p is the droplet velocity, and subscripts 1 and 2 refer to the inlet and exit states of the control volume under consideration.

The quantity $\dot{m}_w$ is related to the total injected water flow rate $\dot{m}_{wt}$ as

$$m_w = m_{wt} \cos \alpha \tag{4}$$

where α represents the angle of injection (measured from the downstream axis).

B. Physical Assumptions

The following physical assumptions are made in the analysis. Many of these assumptions are typically considered in droplet transport models for the theoretical investigation of dispersed droplet dynamics, heat transfer, and combustion, with applications to spray cooling [20,21] and turbulent diffusion of droplets.

1) The pressure in the jet is constant. This approximation is appropriate for perfectly expanded supersonic jets, for which turbulent mixing is a major noise component.

2) The gas and gas-vapor mixture obey perfect-gas law (calorically and thermally perfect). The equation of state for a perfect gas is expressed by

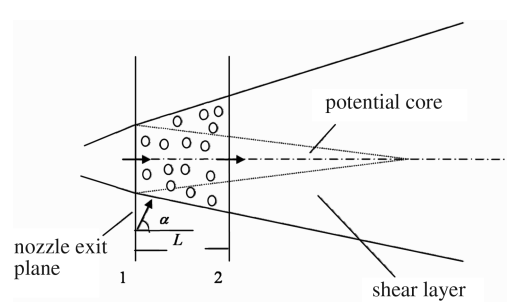

Fig. 1 Schematic of the jet configuration with water injection.

$$\rho_{j1}R_{j1}T_{j1} = \rho_{j2}R_{j2}T_{j2} \tag{5}$$

3) The gas constant for the gas and the gas–water-vapor mixture is approximately the same.

4) The droplets are uniformly distributed in the gas mixture in the control volume.

5) The droplet size is uniform. In reality, the atomization produces a drop size distribution (such as lognormal), and an appropriate characteristic drop diameter (such as Sauter mean diameter d_{32}) may represent drop drag and heat transfer more accurately. The droplet size diminishes as it traverses downstream through the control volume, due to evaporation.

6) The drop temperature is uniform (assumption of infinite thermal diffusivity) and is at the saturation state. Thus,

$$T_p = T_{\text{sat}} \tag{6}$$

The uniform-temperature model belongs to one of the limiting cases of transport processes (Faeth [20]) in evaporating and combusting sprays.

7) Thermal properties of the gas and water are independent of temperature.

8) The droplet drag and heat transfer are approximated by existing semi-empirical relations.

9) The axial extent of the control volume is taken as two jet diameters ($n = L/d_j = 2$) to ensure that the effective jet properties due to water injection are representative. For comparison purposes, the length of the potential core x_t in chemical rockets closely follows the empirical relation of the form (Eldred [22])

$$\frac{x_t}{d_{j1}} = 3.45(1 + 0.38M_{je})^2 \tag{7}$$

where M_{je} stands for the fully expanded jet Mach number. The largest noise-producing region in the jet is known to exist at the end of the potential core. According to [23,24], the maximum intensity of the acoustic sources in a supersonic jet is generally located between 8 and 10 jet diameters.

10) Droplet evaporation is controlled by heat transfer from the gas to the droplets (thermal-conductivity-controlled), as opposed to being diffusion-controlled (Dupays and Vuillot [18]). This assumption appears to be reasonable for the hot jets at relatively high temperatures (typical of rocket exhausts) under investigation. For cold jets, mass transfer by diffusion can become important, with the driving potential represented by the difference between vapor pressure at the droplet surface and the partial pressure of the vapor in the bulk gas.

11) The droplets are assumed to be rigid (no surface-tension effects and no deformation) and spherical.

12) Radiative heat transfer from the drop and the gas is neglected.

13) Parasitic noise generation due to water impact and breakup is not accounted for.

C. Solution

With the preceding assumptions, Eqs. (1–3) and (5) are now solved for the effective jet circumstances at the downstream section 2 of the control volume. A closed-form solution is thereby obtained as follows.

1. *Effective Jet Velocity*

From the momentum equation in conjunction with the continuity equation, an expression of the jet velocity ratio can be obtained as

$$\frac{u_{j2}}{u_{j1}} = \frac{1}{(1 + \eta(\dot{m}_w/\dot{m}_{j1}))}\left[1 - \frac{F_D}{\rho_{j1}u_{j1}^2A_{j1}} + \frac{\eta\dot{m}_w u_p}{\rho_{j1}u_{j1}^2A_{j1}}\right] \tag{8}$$

The second term in the parenthesis on the right-hand side (RHS) can be expressed as

$$\phi = \frac{F_D}{\rho_{j1}u_{j1}^2A_{j1}} = N_pC_D\left(\frac{A_p}{A_{j1}}\right)\frac{1}{2}\left(1 - \frac{u_p}{u_{j1}}\right)^2 \tag{9}$$

where

$$N_p = \frac{3}{2}\left(\frac{\dot{m}_w}{\dot{m}_{j1}}\right)\left(\frac{\rho_{j1}}{\rho_p}\right)\left(\frac{d_{j1}}{d_p}\right)^3 n(1-\eta) \tag{10}$$

denotes the number of droplets per unit volume of the gaseous mixture in the control volume. The quantity d_p stands for the droplet diameter. The quantity ϕ can be expressed as

$$\phi = \psi(\dot{m}_w/\dot{m}_{j1})(1-\eta) \tag{11}$$

where

$$\psi = \frac{3}{2}\left(\frac{\rho_{j1}}{\rho_p}\right)\left(\frac{d_{j1}}{d_p}\right)^3 n\frac{1}{2}C_D\left(\frac{Re_p}{Re_{j1}}\right)^2 \tag{12}$$

The third term on the RHS can be written as

$$\frac{\eta\dot{m}_w u_p}{\rho_{j1}u_{j1}^2A_{j1}} = \eta\left(\frac{\dot{m}_w}{\dot{m}_{j1}}\right)\left[1 - \frac{Re_p}{Re_{j1}}\frac{d_{j1}}{d_p}\right] \tag{13}$$

where Re_{j1} is the jet Reynolds number, and Re_p is the droplet Reynolds number defined by

$$Re_{j1} = \frac{\rho_{j1}u_{j1}d_{j1}}{\mu_{j1}}, \qquad Re_p = \frac{\rho_{j1}(u_{j1} - u_p)d_p}{\mu_{j1}} \tag{14}$$

2. *Effective Jet Temperature*

Consideration of the energy equation in conjunction with the equation of state furnishes the relation for the effective jet temperature:

$$\frac{T_{j2}}{T_{j1}} = \frac{1}{(1 + \eta\frac{\dot{m}_w}{\dot{m}_{j1}})}\left[1 + \frac{u_{j1}^2}{2c_{pj}T_{j1}} - \frac{F_Du_p}{c_{pj}T_{j1}}\right] - \frac{u_{j2}^2}{2c_{pj}T_{j1}} \tag{15}$$

where

$$\frac{u_{j1}^2}{2c_{pj}T_{j1}} = \frac{\gamma - 1}{2}M_{j1}^2 \tag{16a}$$

and

$$\frac{u_{j2}^2}{2c_{pj}T_{j1}} = \left(\frac{u_{j2}}{u_{j1}}\right)^2\left(\frac{\gamma - 1}{2}\right)M_{j1}^2 \tag{16b}$$

3. *Effective Jet Density*

The density ratio is evaluated from

$$\rho_{j2}/\rho_{j1} = T_{j1}/T_{j2} \tag{17}$$

4. *Effective Jet Mach Number*

An expression for the effective jet Mach number is given by

$$\frac{M_{j2}}{M_{j1}} = \left(\frac{u_{j2}}{u_{j1}}\right)\left(\frac{\rho_{j2}}{\rho_{j1}}\right)^{1/2} \tag{18}$$

5. *Effective Jet Exit Area*

The effective jet exit cross-sectional area becomes

$$\frac{A_{j2}}{A_{j1}} = \left(\frac{\rho_{j2}}{\rho_{j1}}\right)\left(\frac{u_{j1}}{u_{j2}}\right)\left(1 + \eta\frac{\dot{m}_w}{\dot{m}_{j1}}\right) \tag{19}$$

D. Droplet Drag and Heat Transfer

1. Droplet Drag

The total drag on the droplet is comprised of both skin-friction drag and form drag. Evaporation reduces the skin-friction drag due to thickening of the boundary layer and alters the form drag by displacement of the point of separation of the boundary layer. Additionally, it introduces property variations in the gaseous phase. Similarity considerations based on boundary-layer theory suggest that the friction coefficient depends on the Spalding transport parameter B, defined by (Kays and Crawford [25])

$$B = \frac{c_{pg}(T_g - T_d)}{h_{fg}(T_d)} \tag{20}$$

Because the magnitude of the form drag is comparable with friction drag, strictly speaking, the drag coefficient in the presence of evaporation cannot be approximated by relations based on boundary-layer-type theories alone (Sirignano [21]). In the present work, the drag coefficient C_D in the presence of evaporation is obtained from a relation due to Eisenklam et al. [26], based on measurements on freely falling drops (water, heptane, pentane, ethyl alcohol, etc.), as

$$C_D = C_{D0}/(1 + B) \tag{21}$$

It was shown in [26] that the factor $(1 + B)$ in Eq. (21) has some theoretical basis with regard to considerations of laminar boundary layer on a body of revolution. This relation has been widely considered, for example, by Mastanaiah and Ganic [27] for two-component air–water dispersed-flow heat transfer and by Sirignano [21] for spray cooling. The preceding equation suggests that the drag coefficient in the presence of evaporation decreases with an increase in the evaporation rate.

The correlation for the drag coefficient in the absence of evaporation C_{D0} appearing in Eq. (21) is obtained from White [28] as

$$C_{D0} = \frac{24}{Re_p} + \frac{6}{1 + Re_p^{0.5}} + 0.4 \tag{22}$$

The effect of Mach number (compressibility) on the droplet drag is not accounted for, however. The first term in Eq. (22) represents the Stokes drag [29] valid for Reynolds numbers less than one.

2. Droplet Heat Transfer and Evaporation

The fraction of the injected liquid that is evaporated is determined as follows. The heat transferred from the gas to the droplets is given by

$$q_p = N_p Nu \pi k d_p (T_{je} - T_p) \tag{23}$$

where N_p is obtained from Eq. (10). The quantity T_{je} is called the *equivalent gas temperature*, which will be discussed a little later.

The drop Nusselt number Nu for an evaporating droplet is computed as follows. With a Couette flow approximation (zero Reynolds number limit), it has been shown that for constant properties, the Nusselt number in the presence of evaporation (mass transfer cooling or transpiration) can be shown to be expressed by (Kays and Crawford [25])

$$Nu = Nu_0 \, \ln(1 + B)/B \tag{24}$$

where Nu_0 denotes the drop Nusselt number in the absence of mass transfer. Strictly speaking, the preceding relation based on boundary-layer-type theory does not hold in the near wake of the droplet (Sirignano [21]). Because most of the heat and mass transport for the droplet Reynolds number of interest (order of 100 or less) occurs before the point of separation, vaporization can be satisfactorily predicted by the preceding relation [21]. This relation has been considered by Mastanaiah and Ganic [27] and Sirignano [21]. The quantity Nu_0 is expressed by the well-known Ranz–Marshall correlation [30]:

$$Nu_0 = 2 + 0.6 Re_p^{0.5} Pr^{0.33} \tag{25}$$

The heat of evaporation for the droplets is provided from

$$q_p = \eta \dot{m}_w h_{fg} \tag{26}$$

Because the droplet is assumed to be at the saturation temperature T_{sat}, heat transfer to the droplet is entirely used to vaporize the droplet (no sensible heating).

From Eqs. (23) and (26), we find an expression for the evaporation fraction η as

$$\frac{1}{\eta} = 1 + \frac{Pr}{Nu} \frac{Re_{j1}}{(d_{j1}/d_p)^2} \frac{1}{(3/2)(\rho_{j1}/\rho_p)n} \left[\frac{h_{fg}}{c_{pj}(T_{je} - T_{\text{sat}})} \right] \tag{27}$$

where Pr is the Prandtl number of the gas (taken as 0.7). Equation (27) suggests that the evaporation factor η is independent of the water-to-jet mass flow rate ratio. This result is believed to be expected, because the droplet concentration is relatively small, such that droplet interaction is negligible.

The equivalent gas temperature T_{je} in Eq. (27) is chosen to ensure that $T_{je} \gg T_{\text{sat}}$ so that the evaporation is heat-transfer-controlled. In the present work, a value of $T_{je} = 1389$ K is considered, which compares with $T_{\text{sat}} = 373$ K (saturation temperature of water at atmospheric pressure). It is tacitly assumed that the chosen value of T_{je} partly accounts for vaporization by mass diffusion.

E. Invariant Groups

An examination of Eqs. (27), (13), and (12) suggests that the following invariant groups appear in the analysis:

$$\frac{Re_{j1}}{(d_{j1}/d_p)^2} \frac{1}{(\rho_{j1}/\rho_p)} = c_1 \tag{28}$$

$$\frac{Re_p}{Re_{j1}} \left(\frac{d_{j1}}{d_p} \right) = c_2 \tag{29}$$

and

$$\left(\frac{\rho_{j1}}{\rho_p} \right) \left(\frac{d_{j1}}{d_p} \right)^3 \frac{Re_p}{Re_{j1}^2} = c_3 \tag{30}$$

Because

$$c_2 = c_1 c_3$$

it is evident that two independent relations exist for the estimation of Re_p and d_{j1}/d_p. These are now evaluated as

$$\frac{d_{j1}}{d_p} = \left[\frac{1}{c_1} \frac{\text{Re}_{j1}}{(\rho_{j1}/\rho_p)} \right]^{1/2} \tag{31a}$$

$$Re_p = c_2 \frac{Re_{j1}}{(d_{j1}/d_p)} \tag{31b}$$

Thus, the unknown quantities Re_p and d_{j1}/d_p are determined in terms of the adjustable constants c_1 and c_2. In view of the complexity with regard to the knowledge of droplet diameter and drop Reynolds number, it is postulated here that the constants c_1 and c_2 are *invariant*. These constants may be established by a correlation with available test data.

F. Drop Characteristics

In view of the critical role of the drop size in the present investigation, we briefly present a review of the physical processes that control the drop characteristics in systems involving sprays and atomization (Sirignano [21]).

1. Drop Deformation and Drop Breakup

The physical processes of drop deformation and breakup are extremely complex (Hsiang and Faeth [31]. According to the classical description of drop breakup, atomization occurs by primary breakup near the liquid surface, followed by secondary breakup. Two types of primary drop breakup are generally observed: bag breakup (deformation initiated near the upstream end of the drop) and shear breakup (liquid shearing at the periphery of the drop). Multimode breakup is also present. The breakup processes are not instantaneous.

Hinze [32] demonstrated that the primary breakup-regime transitions are primarily dependent on the Weber number We and the Ohnesorge number Oh, defined by

$$We = \rho|u_d - u_g|^2 d_p/(2\sigma), \qquad Oh = \mu_d/(\rho_d d_p \sigma)^{1/2} \tag{32}$$

The secondary breakup is correlated in terms of the Eotos number E_0 (also known as Bond number Bo), defined by [31]

$$Eo = a_d \rho_d d_0^2/\sigma \tag{33}$$

where a_d denotes the drop acceleration. Results show that the secondary breakup is not a localized process, but extends over a significant region of about 40 drop diameters.

Measurements suggest that the critical Weber number We_{cr} required for the onset of drop breakup (transition criterion for breakup under steady disturbances) is given by (Faeth [20] and Hsiang and Faeth [31]

$$We_{\mathrm{cr}} = Re_{\mathrm{cr}}^{1/2}/2 \tag{34}$$

where Re_{cr} is the critical Reynolds number of the drop.

2. Drop Size

On account of the shearing action between the gas and the drop, drops break up quickly from their initial size near the injector station to a few microns downstream of the injection station (Krothapalli et al. [8]). Turbulence in the gas phase could be influenced by the presence of the droplets. Turbulence in the gas phase is generally reduced by the injection of droplets [8]. Measurements of Krothapalli et al. [8] suggest that the arithmetic mean diameter remains fairly uniform across the jet.

G. Jet Mixing Noise Reduction

Once the effective jet conditions (velocity, temperature, and Mach number) are obtained, the reduction in the jet turbulent mixing noise is evaluated with the aid of the scaling laws proposed by the author [33,34]. In these references, the turbulent mixing noise reduction is presented as a function of the jet Mach number and the jet-to-ambient-temperature ratio (see Fig. 2) in both subsonic and supersonic flows. The scaling laws are validated with test data in [34]. A brief account of the scaling laws and the associated assumptions on which the mixing noise reduction has been evaluated is briefly summarized as follows.

Based on dimensional arguments and the theories of Lighthill [35,36], Ffowcs-Williams [37], and Ribner [38] and on experimental data on cold and hot jets in subsonic and supersonic flows, a scaling law for the mean square sound pressure for subsonic and perfectly expanded supersonic jets is as follows [33,34]:

$$\frac{\overline{p^2(\theta, f)}}{(\rho_j u_j^2)} = K_1 \left(\frac{\rho_\infty}{\rho_j}\right)\left(\frac{u_j}{c_j}\right)^{3.5}\left(\frac{c_j}{c_\infty}\right)^{3.5} G_1(M_c, \theta) G_2\left(\frac{f}{f_p}, M_c, \theta\right) \tag{35}$$

where the quantity K_1 is a proportionality constant, G_1 is the directivity factor (owing to source convection), G_2 accounts for the spectral distribution of the sound power, the quantity M_c refers to the convective Mach number (related to the jet Mach number), θ is the angle from the downstream jet axis, and f_p is the peak frequency. The effect of jet temperature is accounted for by Lighthill's [36] suggestion through the changes in the density factor in the quadrupole noise field.

The directivity factor is essentially the same as that provided by the Lighthill–Ffowcs-Williams–Ribner formulation, given by

$$G_1 = [(1 - M_c \cos\theta)^2 + \alpha M_c^2]^{-5/2} \tag{36}$$

where $\alpha = 0.4$ is considered. A generalized similarity spectrum (semi-empirical) is also proposed to account for both the fine scale structure and the turbulence structure associated with Mach wave radiation as

$$G_2 = \frac{(f/f_p)^{5/4a}}{[1 + (f/f_p)^2]^{9/6a}} \tag{37a}$$

where

$$a = \{0.2 + \exp[-2M_c/\sin(\theta/2)]\}^{0.35} \tag{37b}$$

In the author's recent work [34], the scaling laws proposed were validated with test data for the overall sound power level (OAPWL), OASPL, directivity, and spectral content over a wide range of temperatures and jet Mach numbers. Figure 2 shows that at a given M_j, the OAPWL depends only on the temperature ratio. For convenience, the data are plotted with reference to OAPWL at $M_j = 1$ and $T_j/T_\infty = 1$ (isothermal jet). As is to be expected, the OAPWL transitions from a u_j^8 dependence in subsonic flow to u_j^3 at large supersonic Mach numbers. Referring to the isothermal jet, an increase of sound power of about 56 dB is predicted as the jet Mach number is increased form 0.2 to 1.0. Although the calculations are shown for jet Mach numbers of up to 10, they should be viewed with caution for jet Mach numbers in excess of about 5 (hypersonic flow), for which real-gas effects (including dissociation) can become significant.

According to the early work of Lush [39] and Fisher et al. [40], there are drastic differences between the predictions based on Lighthill–Ffowcs-Williams formulations and the test data. There exists considerable debate on several aspects with regard to the differences between cold and hot jets; this situation is so in spite of the existence (for two or three decades) of theories of jet noise accounting for temperature effects, which can be improved with reference to the more recent experimental data. The development of computational aeroacoustics, perhaps in conjunction with large eddy simulation [41–43], can be a further tool in understanding the differences between cold and hot jets.

With regard to the spectral distribution of turbulent mixing noise reduction, the author [44] has proposed an extension of the theory of Temkin and Dobbins [17] for dilute suspensions with nonlinear particle relaxation (particle momentum and heat transfer). Satisfactory comparison is shown between the prediction and the data of Norum [7] for the spectral attenuation of sound.

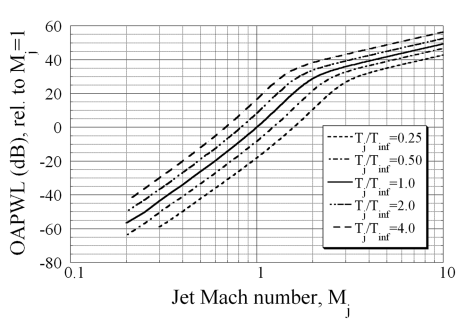

Fig. 2 Effect of jet Mach number and jet temperature on overall sound power, from Kandula [34].

III. Results and Comparison

The preceding analysis suggests that the effective jet conditions in the presence of water injection are dependent primarily on the water-

to-gas mass flow rate ratio and are independent of the momentum-flux ratio, which primarily governs the penetration depth of the injected water normal to the jet. A correlation of the analysis with the test data of Norum [7] for hot supersonic turbulent mixing noise reduction at $M_j = 1.45$ yields the values of the invariants c_1 and c_2 as

$$c_1 = 5, \qquad c_2 = 0.05 \tag{38}$$

Thus, in all the results presented subsequently, the preceding values of the invariants are considered. Details of the evaluation of these constants are provided in Sec. III.B.

A. Predictions for the Effective Jet Conditions

Figure 3a displays the variation of fractional evaporation η as a function of the jet exit Reynolds number for various jet exit temperatures, as given by Eq. (27). It is seen that evaporation increases with an increase in jet Reynolds number and an increase in jet temperature, as is to be expected. The variation of η with the jet temperature is illustrated in Fig. 3b.

Figure 4a shows the variation of the effective jet velocity with the water mass flow rate ratio at $Re_{j1} = 10^7$. The effective jet velocity decreases with an increase in water flow rate on account of the momentum transfer between the gas and the droplets. For mass flow rate ratios beyond about four, the rate of decrease in the jet velocity is relatively small. This result seems to be consistent with the experimental observation of Zoppellari and Juve [5], which suggests that beyond a mass flow rate ratio of about four, further reductions in jet noise are not appreciable.

The variation of the effective jet temperature with the water flow rate is displayed in Fig. 4b for various jet exit Mach numbers at $Re_{j1} = 10^7$. The effective jet temperature decreases with an increase in water flow rate due to heat transfer from the gas to the droplets and subsequent evaporation. The rate of decrease in the effective jet temperature begins to slow for water flow rate ratios beyond about four, as in the case of the effective jet velocity. At a given mass flow rate ratio, the effective jet temperature decreases with an increase in the jet exit Mach number. A crossover trend with the exit Mach number is noted at a flow rate ratio of about 0.6. This trend may be physically explained as follows. At a low water mass flow rate ratio, the denominator in the first term of Eq. (15) remains close to unity, so that at a given Re_{j1}, the ratio T_{j2}/T_{j1} increases as M_{j1} increases. On the other hand, for relatively large values of water mass flow rate ratio, the drag-related term becomes appreciable, so that at a given Re_{j1}, the ratio T_{j2}/T_{j1} decreases with increasing M_{j1}.

Illustrated in Fig. 4c is the dependence of the effective jet Mach number as a function of the water mass flow rate at $Re_{j1} = 10^7$. The results suggest that below a water flow rate ratio of one, the effective jet temperature is almost independent of the jet exit Mach number. This trend is to be expected because at low water mass flow rate ratios less than about one, the changes in the gas temperature across the control volume are not appreciable relative to the velocity changes (see Figs. 4a and 4b). Thus, the effective jet Mach number is primarily dependent on the effective jet velocity, which in turn is independent of the jet exit Mach number.

Figure 4d depicts the distribution of the effective jet density with the water mass flow rate at various jet exit Mach numbers at $Re_{j1} = 10^7$. It is seen that the effective jet density increases with the flow rate and with an increase in the jet exit Mach number. Calculations suggest that the jet cross-sectional area increases with the water flow rate (Fig. 4e). At a fixed flow rate ratio, the effective jet area decreases with an increase in the jet exit Mach number.

The effects of jet exit Reynolds number on the distribution of effective jet velocity, jet temperature, and jet Mach number are indicated in Figs. 5a–5c. Figure 5a suggests that the change in the effective jet velocity is relatively small for a three-order-of-magnitude change in the jet exit Reynolds number. A similar trend is noted with regard to the dependence of jet exit temperature and jet Mach number on the jet exit Reynolds number (Figs. 5b and 5c).

These results for the variation of the effective jet conditions suggest that the theory of invariant groups formulated here (in terms of the constants c_1 and c_2) seems to provide a satisfactory first step in our understanding of the role of water injection in reducing the jet velocity and jet temperature.

It should be cautioned that the present results would entail error at a relatively high water flow rate in view of the parasitic noise generation due to water impact.

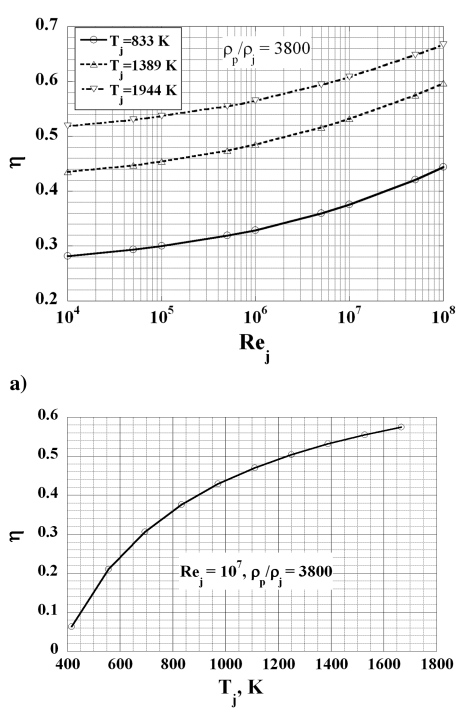

Fig. 3 Variation of the fractional evaporation with the a) jet Reynolds number and b) jet temperature.

B. Comparisons with Experimental Data

A comparison of the present theory with the test data of Norum [7], from which c_1 and c_2 are determined, is presented in Fig. 6. The data correspond to the hot supersonic jet of air from a convergent–divergent nozzle operation at $T_t = 867$ K, and $M_j = 1.45$. The jet exit Reynolds number Re_{j1} is about 1.3×10^6. At this condition, supersonic jet mixing noise dominates the upstream noise radiation, and Mach wave radiation dominates the downstream noise radiation. In the data, water is injected at 45 deg, and the number of injectors includes 6, 12, and 18. The data shown correspond to maximum noise reductions with water injection. There is scatter in the data for OASPL reduction at a given water mass flow rate ratio. Thus, the data shown here represent averaged values.

In the data of Norum [7], acoustic measurements are made for angles θ between 30 and 135 deg to the jet downstream axis (a total of 28 microphones, evenly distributed). Data for the maximum noise reduction with mass flow rate ratio are presented. The reduction is more or less evenly distributed across the frequency, with a maximum reduction of 3.8 dB achieved in the upstream direction (of 135 deg) and with about 1 dB reduction accomplished in the peak-radiation direction. The spectra and directivity of OASPL reductions are shown only for the highest mass flow rate ratio. Thus, the test data correspond primarily to 135 deg to the jet downstream axis. The correlation of the present theory with the data for the evaluation of the constants c_1 and c_2 is carried out for the upstream direction and *only at the highest mass flow rate*. These constants are considered for the estimation of noise reduction at other flow rates, and the validity of the theory is thereby tested.

For the test conditions considered previously, the present theory suggests that $\eta = 0.49$, indicating that 49% of $\dot{m}_w$ is evaporated within the control volume chosen. At these conditions, the present theory yields $Re_p = 2.1$ and $d_{j1}/d_p = 3.13 \times 10^4$, as obtained from

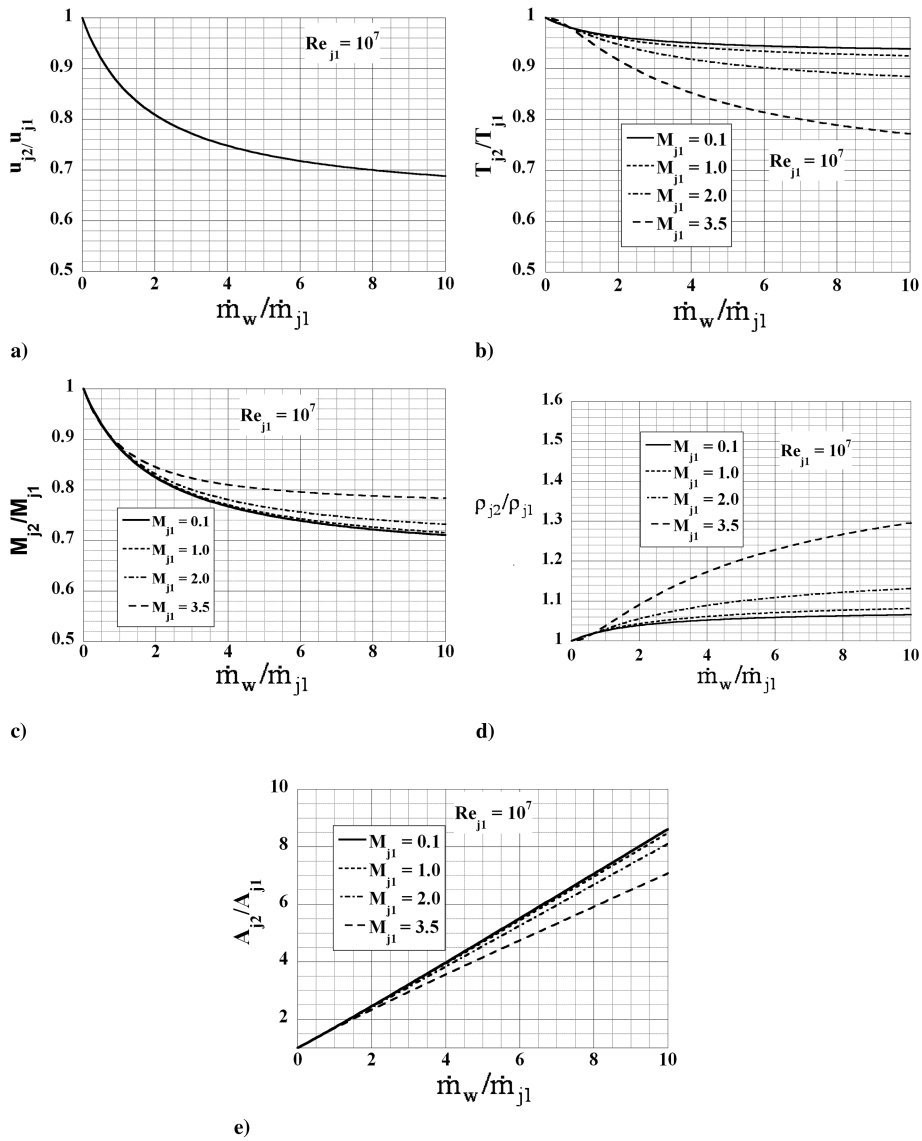

Fig. 4 Variation of effective jet a) velocity with the water mass flow rate, b) temperature with the water mass flow rate, c) Mach number with the water mass flow rate, d) density with the water mass flow rate, and e) cross-sectional area with the water mass flow rate.

Eqs. (31a) and (31b). The agreement between the present prediction and the data is seen to be favorable, considering the complexity of the problem and the number of simplifying assumptions made in the analysis. Both the data and the theory suggest that the reduction in mixing noise increases with an increase in water flow rate. At the highest water flow rate considered in the data, a reduction of 3.8 dB in the mixing noise is achieved. Calculations show that the noise reduction is primarily achieved by jet velocity reduction through momentum transfer, whereas the jet temperature reduction is small enough to contribute appreciable noise reduction.

IV. Discussion

The invariants postulated here are applicable to the control volume only and are not applicable to the entire flowfield, because the shear layers dominate the flowfield relative to the potential core. This paper is concerned primarily with conditions in the neighborhood of the nozzle exit with regard to the determination of the effective jet conditions. The author believes that the assumptions and the invariant postulate are reasonable and valid. The principal objective of the present work is to understand the mechanisms of noise reduction with regard to the particulate relaxation processes. A sophisticated numerical analysis based on Navier–Stokes computational fluid dynamics and computational aeroacoustics [41–43] would be helpful but is outside the scope of the present study.

A distinction could be made between water-injection systems and microjet injection as follows. Experimental data of [8–10,16] with microjets and the data with a higher mass flow rate ratio (such as those of Norum [7], in conjunction with the recent theory) suggest that a water mass flow rate ratio of about 0.05 to 0.1 is representative of the boundary between water microjet injection and water injection. Water microjet injection reduces jet noise by reducing turbulent jet mixing noise and mitigating Mach wave radiation, which is regarded as the momentum-ratio effect, as observed by the measurements of Greska and Krothapalli [9]. This momentum-ratio parameter J is defined by

$$J = \frac{\rho_{\mathrm{mj}} u_{\mathrm{mj}}^2}{\rho_j u_j^2} \tag{39}$$

where the subscript mj refers to the microjet.

Microjets have been shown to result in a moderate noise reduction, primarily due to the reduction of turbulence levels and the alteration of the shock structure (and the associated shock–turbulence interaction) (see Krothapalli et al. [8]). The data of Greska and Krothapalli [9] indicate that in the presence of shock waves, the noise reduction with the aid of microjet injection could be satisfactorily correlated by the momentum-ratio parameter.

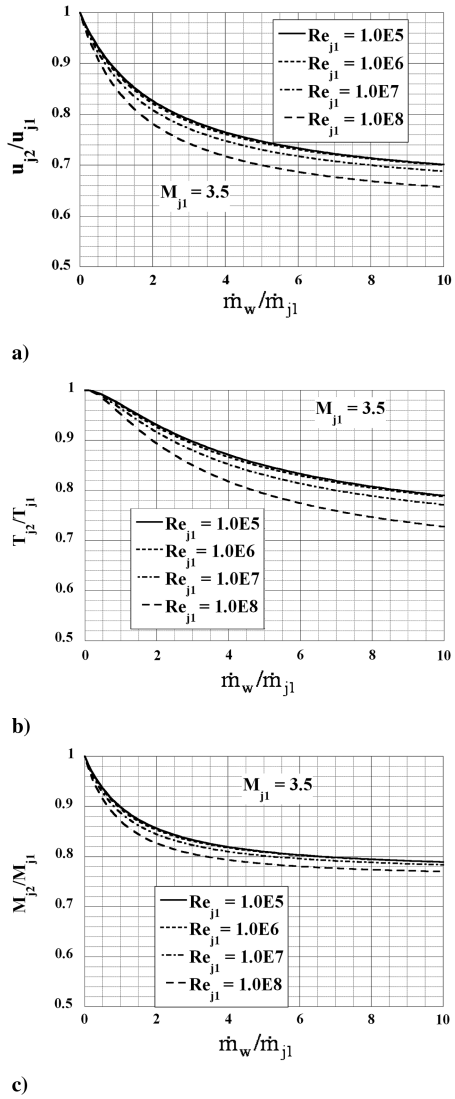

Fig. 5 Dependence of effective jet a) velocity with jet Reynolds number, b) temperature on jet Reynolds number, and c) Mach number on jet Reynolds number.

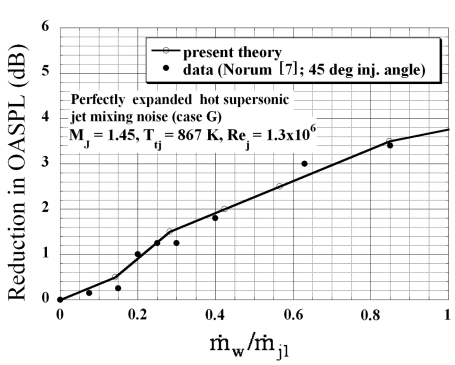

Fig. 6 Comparison of the predictions with data of Norum [7] for turbulent mixing noise reduction due to water injection.

The manner in which the region covered by the control volume (covering only the first two downstream diameters) affects the turbulence at the end of the potential core (which, according to Eq. (7), is approximately 8.3 diameters downstream for the $M_j = 1.45$ example considered) is explained as follows. Measurements by Krothapalli et al. [8] in a Mach 1.4 nozzle suggest that the presence of water droplets in the jet modified the turbulence structure significantly, resulting in axial and normal rms velocity fluctuations (transverse turbulence intensity) of about 10 and 30%, respectively, relative to a normal jet (without mass injection). At the low mass flow rate ratios of about 0.05 considered in this reference, the mean flow structure (velocity field) remains essentially unaltered by the water injection. Under these circumstances of microjet injection (say, at a mass flow rate ratio below about 0.05), it is believed that the turbulence reduction due to the breakup of water droplets and the presence of the dispersed phase are responsible for noise suppression. The noise-reduction levels are spread over all directions (including the peak-radiation direction for Mach wave radiation) and all frequencies.

It is thus seen that for water microjet injection, noise reduction is manifested primarily through a reduction of jet turbulence levels. For water injections at relatively high mass flow rate ratios (say, ratios in excess of about 0.05 to 0.1) under consideration in the present work, the jet mean flow structure (velocity and temperature) is changed by the presence of the droplets and their interactions (momentum and heat transport). The attendant changes in turbulence levels and noise modification thereof are perhaps not appreciable because the mean flow velocity is simultaneously diminished by the injection process. Energy from the mean flow to the turbulence may actually be reduced under these circumstances. It should be pointed out, however, that a detailed account of turbulence modification in the source region due to the droplet presence and interaction with regard to noise reduction is beyond the scope of the present work.

It is to be remarked that the present theory, asserting that the noise reduction may be estimated in terms of two dimensionless invariant groups, has to be confirmed in the future by further comparisons with experimental data.

V. Conclusions

The effect of water injection on turbulent jet mixing noise reduction has been theoretically investigated on the basis of a one-dimensional control volume formulation for the effective jet exit conditions. The theory yields two dimensionless invariant groups, involving the ratio of droplet diameter to jet exit diameter and the ratio of droplet Reynolds number to jet Reynolds number. Correlation of the theory with available data for hot supersonic jet mixing noise reduction provided a satisfactory agreement. The theory predicts the experimental trend that the jet mixing noise reduction increases with an increase in the water mass flow rate. It is demonstrated that above certain mass flow rate ratios (typically exceeding microjet injection levels), the water mass flow rate ratio is an important parameter characterizing the mixing noise reduction. The conception of effective jet exit conditions proposed herein appears to be a significant first step in our understanding of the mechanisms of jet noise reduction due to water injection.

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